

# THE SQUARE KILOMETER ARRAY: NEW CHALLENGES FOR COSMOLOGY

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*To appear in:*

“The Westerbork Observatory; Continuing Adventure in Radio Astronomy”  
*Eds. E. Raimond & R. Genée (Kluwer, Dordrecht)*

## 1. Introduction

In the past three decades our view and understanding of the structure and evolution of the universe has grown in an almost revolutionary fashion. The discovery of the rich variety of patterns displayed by the galaxy distribution, the discovery of streaming motions in the local universe, the detection of tiny variations in the temperature of the microwave background radiation and the use of powerful computational resources for simulating the formation and evolution of structure are some of the most conspicuous developments that transformed the field of cosmology into one of the most active branches of astrophysics<sup>1</sup>. It is therefore plausible that the Holy Grail of cosmology – the answer to the question of how the wealth and variety of structure in our cosmos rose out of an almost featureless, extremely hot and dense early universe – may soon come within reach.

Twenty-five years ago, the prospect of measuring the structure of the universe was one of the main motivations for building the Westerbork Synthesis Radio Telescope. In particular, it was hoped that by counting radiosources as a function of their brightness it would be possible to determine the cosmological parameter  $q_0$ , which measures the deceleration of the expansion of the universe. That this expectation has not been fulfilled was due to several factors. The main reason was that the surveyed volume did not quite reach out to the large redshifts necessary for discriminating between different values of  $q_0$ . Moreover, the interpretation of the data

<sup>1</sup>for up-to-date discussions and overviews on cosmology see e.g. Kolb & Turner 1990; Peebles 1993; and Jones 1994; excellent texts that focus specifically on the issue of large scale structure and structure formation are Padmanabhan 1993 and Coles & Lucchin 1995; Peebles 1980 is the standard reference.

got insuperably complicated by the rich variety of radiosources and their evolution. In combination with fascinating discoveries such as radiosources with jets this lead to a shift of research interests. In the developments in the subsequent golden era of cosmology radio observations therefore hardly staged more than a background role, although quite often a significant one.

With the upgrades of the Westerbork Telescope (WSRT) and the VLA, and in particular when the ambitious plans for a Square Kilometer Array become reality, it is quite likely that radio astronomy will play a keyrole in the cosmological developments of the coming decades. This contribution <sup>2</sup> is meant to give an impression of some of the main cosmological issues in which the SKA could play a large role and to whose solution the SKA could contribute significantly. Firstly, in section 2 we will give a general inventory of possible cosmological applications, some of which we will discuss in more detail. Of great interest is the use of the SKA for carrying out redshift surveys on the basis of the 21 cm line. In section 3 we will therefore focus in particular on this aspect of galaxy redshift surveys, and describe in more detail what the present knowledge of the morphology and geometry of the galaxy distribution is, and why the SKA would be such a magnificent instrument for mapping the galaxy distribution to high redshifts, and therefore to provide important contributions towards understanding the formation of structure in the universe. In the two subsequent sections we discuss the technical issues involved with the use of the SKA for the described cosmological applications. In particular we will address the number of gas-rich galaxies that we can hope to detect and the observing time required to detect a given amount of hydrogen gas at some redshift  $z$ . On the basis of these considerations we then conclude with specifying the requirements that would make the Square Kilometer Array into an ideal instrument for cosmology.

## **2. The Square Kilometer Array: Inventory of its cosmological potential**

It have been mainly optical telescopes that were instrumental in mapping the large-scale structure of the galaxy distribution. By virtue of the Hubble expansion of our universe we can determine the distance of each galaxy simply by measuring the redshifts of the spectral lines in their electromagnetic spectrum. For the large majority of galaxies this has been done for lines in the optical region of their spectra. As yet the potential of determining the redshifts of galaxies with radiotelescopes – from the 21 cm (1420 MHz) line

<sup>2</sup>We should note here that this contribution is not intended to be an extensive review, and we therefore apologize in advance for failing to cite all relevant and important contributions to the items discussed.

emitted by neutral atomic hydrogen (HI) – has not been explored and applied over a large volume of the universe. This is mainly due to the limited frequency coverage and insufficient sensitivity of present-day radio emission receivers.

In the context of galaxy redshift surveys it is important to know that in the local universe essentially all neutral atomic hydrogen is associated with galaxies. There is no evidence for the existence of a major intergalactic component of HI at low redshifts, although this situation may have been considerably different at earlier epochs. At higher redshifts a large fraction of the hydrogen gas may actually have resided in intergalactic cloud complexes whose content accreted upon (proto)galaxies. On the other hand, because their stars have formed out of their supply of hydrogen gas, the hydrogen content of galaxies must have been larger in the past than it is now. Another significant point of 21-cm line redshift surveys is that they concentrate specifically on spiral and irregular galaxies. These late-type galaxies contain substantial amounts of HI gas and are therefore easily detectable by radio telescopes. On the other hand, radio telescopes can not be used to detect and map the distribution of elliptical and SO galaxies, as they do not contain enough atomic hydrogen. These early-type galaxies form only a small minority of the total galaxy population, residing preferentially in high-density regions such as clusters of galaxies. The spiral and irregular galaxies, representing the major share of the galaxies in the universe, constitute a considerably less biased tracer of the distribution of (luminous) matter, and they can be found in high as well as low density regions. Moreover, an important general property of these late-type galaxies is that their relative atomic hydrogen content increases with decreasing optical luminosity (see e.g. Briggs 1990). As a result, ‘dwarf’ galaxies are far easier to detect with radio telescopes than with optical telescopes. Because the number of dwarf galaxies far exceeds the number of giant galaxies, the mapping of their distribution is essential for obtaining a complete picture of the large scale distribution of galaxies in space. Radio redshift surveys therefore not only provide a useful and efficient additional way of measuring galaxy redshifts, for the purpose of ‘cosmography’ they also form a necessary complement to the optical redshift surveys.

So far most 21-cm spectroscopy work has been carried out with single-dish telescopes. In particular the Arecibo antenna has been instrumental in obtaining 21-cm line redshifts, of which by now there have been measured some 12,000 (see Giovanelli & Haynes 1991). Single-dish radio telescopes, however, cannot compete against radio interferometric instruments in spatial resolution and efficiency. In this context it is worthwhile to emphasize the interesting advantage that redshift surveys with radio aperture synthesis instruments will have over their optical counterparts. At optical wave-

lengths, redshifts are basically determined ‘galaxy by galaxy’, after they have been identified on the sky. The same is true for redshift determinations with single-dish radio telescopes. With multi-dish radio interferometer telescopes on the other hand, a predefined region of the sky gets ‘mapped’ in small frequency bands that together cover a wide frequency range. This spectral ‘mapping’ capability of such radio telescopes is the key to a highly efficient method of determining redshifts - making it possible to obtain the 21-cm line profiles and redshifts for huge numbers of galaxies in the same observation. The coming upgrade of the receivers of the Westerbork array (WSRT) in 1997 will therefore undoubtedly lead to a significant improvement of the radio redshift survey capacity and enable comprehensive radioastronomical studies of the structure and kinematics of the local universe.

An even more dramatic addition to our understanding of the morphology and evolution of structure in the universe can be expected from the construction of a Square Kilometer Array (SKA). As this instrument will surpass any existing radio telescope by two orders of magnitude in sensitivity it would be difficult, taking into account the historical background, to try to predict how radio cosmology based on such an advanced instrument would develop in the next decades. It would even be difficult to aim at a complete and detailed scientific justification for such an instrument. Nonetheless, given this caveat, it is of course a challenging exercise to contemplate what an instrument like the SKA might contribute to the study of the large-scale structure of the universe.

In this contribution we will outline in more detail the cosmological background and issues for which the SKA promises to be such an extremely powerful and important instrument. Indeed, we will argue that observations with the SKA will often be essential for significant advances in studying the global properties of the universe as well as its ‘large scale structure’ on megaparsec to gigaparsec scales. In this context we will in particular focus on the possibility of studying the neutral hydrogen gas in and around (and in between) galaxies out to large redshifts. Gas masses characteristic for galaxies similar to our own will probably be detectable out to redshifts of around  $z \sim 3$ . This will most likely be a fairly sharply defined redshift cut-off. It would demand too much effort to attempt to detect neutral hydrogen beyond this redshift threshold, because the expansion of the universe leads to an even stronger decrease of brightness with distance than the usual geometric fall-off. But inside that enormous volume hundreds of millions of galaxies can in principle be detected, which will allow a detailed cartography of the large and smaller scale features in the distribution of luminous matter as well as an accurate diagnosis of the value of the global cosmological parameters. Throughout we will make the implicit assumption that

the evolution of the gas clouds will be not so substantial that they would severely degrade the cartographical capacity of the SKA. In particular, we will assume that ionisation of hydrogen by quasars at high redshift will not lead to a drastic reduction of the amount of neutral hydrogen. In this respect it is comforting that absorption line studies of quasar spectra indicate that there are large numbers of HI clouds at high redshifts.

The fact that an instrument like the SKA would enable us to chart out the distribution of essentially *all* late-type galaxies over the largest part of the visible universe implies that it would also allow us to study the *evolution* in the structure of the universe, over a time interval corresponding to 80 or 90 per cent of its age. Mapping out galaxy positions to presently almost inconceivable redshifts of  $z \sim 2.5\text{--}3.0$  means that we will have a look at the structure of the universe when it was only some 2 billion years old, compared to its present age of about 12 billion years. Besides redshift surveys, also other cosmologically relevant observations will greatly benefit from the SKA. One such key observation is the mapping of the velocity field in the local universe, of crucial importance in obtaining a better understanding of the dynamics underlying the structure formation process. Reasonably accurate estimates of the peculiar velocities of spiral galaxies with respect to the expanding universe can be obtained via the Tully-Fisher relation, which links a spiral galaxy's rotation velocity to its luminosity. With the help of the SKA it will be possible to considerably extend the region in which one can measure these peculiar velocities. This would for example open up the exciting possibility of studying in detail the dynamics of structural features like 'filaments', 'great walls' and 'voids'.

Another issue on which the SKA will shed new light is that of the formation and evolution of galaxies. The 21-cm line profile not only yields information on the redshift of these galaxies, but also on their structure and kinematics. By virtue of the imaging capabilities of the SKA, coupling high sensitivity to a good resolution, it will be possible to investigate the kinematics and morphology of gas-rich galaxies out to high redshifts,  $z \sim 1$  and probably even higher. At a redshift as high as 4 the first galaxies may well have formed already, that is, individual gas clouds may have converted part of their hydrogen gas into stars. The SKA will make it feasible to study the changes in the atomic hydrogen content of the galaxies due to the formation of stars and other astrophysical processes, like the formation and evolution of galaxy discs, over a time period of nearly 10 billion years, i.e. over almost the entire history of the universe (except of course for the crucial first few billion years). The SKA will therefore not only allow a study of the evolution of the large-scale distribution of galaxies but also a (statistical) study of the evolution of galaxies themselves.

Observations with the SKA may also lead to important contributions

towards an accurate and unbiased determination of the global cosmological parameters. Better estimates of the expansion rate of the universe – the Hubble parameter<sup>3</sup>  $H_0$  and  $q_0$  – will be possible through the application of the Tully-Fisher relation to galaxies at great distances. Moreover, if the SKA would be provided with receivers in the Gigahertz spectral region it would be an ideal instrument for imaging the Sunyaev-Zel’dovich effect – the distortion of the microwave background spectrum due to the Compton scattering of microwave background photons by hot gas in galaxy clusters. Estimates of the value of the Hubble parameter, as well as that of the cosmic deceleration parameter  $q_0$ , follow out of the combination of these observations with X-ray measurements of the same clusters. They have the important advantage that they are not plagued by contaminating effects of locally induced peculiar velocities. Another interesting possibility of determining the Hubble parameter via radio astronomical work involves the search for gravitationally lensed radio ring images of background extended radio sources in a suitably large and deep radio survey. These rings are a consequence of the general relativistic effect of gravitational focussing of light by a mass concentration like a cluster of galaxies. As the configuration that leads to the formation of radio rings is so highly constrained, it would be possible to infer a value of  $H_0$  without too many underlying assumptions. By applying classical cosmological tests like source counts, more so than via the Sunyaev-Zel’dovich effect, the SKA is expected to be of great value in measuring the rate at which the expansion of the universe slows down. It is quite feasible that radio source counts with the SKA – in particular involving galaxy samples of which also the redshift can be determined and of which the evolutionary properties are known through the imaging capabilities of the telescope – will go down to such faint flux levels (flux = observed brightness) that it finally will become possible to determine  $q_0$ , and therefore the matter content of the universe, quantified by  $\Omega_0$ . It may even be possible to get a reliable estimate (or upper limit) to the cosmological constant  $\Lambda$ .

### 3. Cosmological Cartography: the large scale structure and morphology of the local universe

When averaged over enormous scales of hundreds of megaparsecs the universe appears to be rather uniform. On smaller scales on the other hand, the salient features displayed by the distribution of galaxies testifies of another story (e.g. fig. 1). We see a baffling variety of structures over a wide range of scales within the limited realms of the local universe wherein structures have been mapped in some detail. Galaxies tend to clump in a hierarchy of

<sup>3</sup>in this contribution we write the Hubble constant to be  $100h$  km/s/Mpc

ever larger agglomerations, ranging from small groups of a few dozen galaxies up to rich clusters of galaxies containing several hundreds to thousands of them, which in their turn often group in superclusters that sometimes house several tens of clusters and many more small groups.

Whether the wealth of structure observed in the distribution of galaxies also reflects the structure of the underlying matter distribution is a not yet completely settled issue. But if we wish to use the galaxy distribution to infer information on the large scale matter distribution and the structure formation process we must make that assumption. Fortunately, observations have indicated it to be plausible.

### 3.1. GALAXY REDSHIFT SURVEYS

A first requisite for studying the galaxy distribution is to map their positions over a large part of the universe. A lot of information on the clustering of galaxies can already be obtained from sufficiently well-defined surveys of the galaxy distribution on the sky. They have the advantage that it is relatively straightforward to map the positions of a huge number of galaxies. In particular noteworthy in a historical context is the famous Lick map constructed by Shane & Wirtanen (1967), covering nearly the whole sky and containing around a million galaxies with an apparent magnitude  $m < 19$ . Even better and more objectively defined surveys have recently been obtained by using automatic plate-measuring machines. In particular the Edinburgh-Durham Southern Galaxy Catalogue (Heydon-Dumbleton, Collins & MacGillivray 1989) and the APM catalogue (Maddox et al. 1990a; Maddox, Efstathiou & Sutherland 1990b) have been very useful for cosmological research. For example, the APM catalogue contains about two million galaxies with  $m < 20.5$  located in a region of 4300 square degrees on the southern sky, probing to an effective depth of around  $600h^{-1}$  Mpc.

However, the sky catalogues do not contain distance information, and are therefore very limited when one addresses the issue of the patterns in the galaxy distribution. Fortunately, the expansion of the universe provides us with a good and easy to measure estimate of a galaxy's distance, the redshift  $z$  of the electromagnetic spectrum of the light emitted by the galaxy due to its recession velocity  $V_0$ . In the first half of this century Hubble discovered that the velocity  $V_0$  with which an object speeds away from us is directly proportional to its distance  $r$ ,  $V_0 = H_0 r$ . The precise value of the expansion rate – the Hubble parameter  $H_0$  – is not yet known, although significant progress has recently been made on the basis of observations with the Hubble Space Telescope (on the basis of observations of Cepheids in M100 and M96  $H_0$  has been estimated to be in the order of 70 – 80 km/s/Mpc, Freedman et al. 1994). In addition, one should realize that the

redshift of a galaxy not only relates to its cosmic expansion velocity but that it also includes its peculiar velocity with respect to the expanding background universe (see next section). Although the latter usually represents only a minor contribution, redshifts should be taken merely as first estimates of distance. Because it is relatively easy to determine redshifts, with the arrival of very efficient spectrographs and multiple object spectrographs it became feasible to chart large areas of the local universe. For statistical studies it is important to use well defined galaxy samples. Usually the survey objects are identified from sky surveys on the basis of some selection criterion, often the apparent brightness determined from optical sky survey plates, but also for example the infrared or X-ray flux measured with a satellite. To get an idea of the patterns in the large scale galaxy distribution, and their geometry, the redshift surveys should also subtend a sufficiently wide solid angle on the sky. Such systematic wide angle redshift surveys have become our primary source of information for these purposes. Although early redshift compilations like the one by Humason, Mayall & Sandage (1956) and the Revised Shapley-Ames Catalogue of nearby Galaxies (Sandage & Tammann, 1981) already contained a wealth of information on structures in the nearby universe, the compilation of comprehensive wide angle redshift surveys really started in the 1980s with the famous Harvard-Smithsonian Center for Astrophysics (CfA) survey by Geller, Huchra, and collaborators (see e.g. Geller & Huchra 1989). In the meantime numerous other surveys have become available, and some very ambitious optical surveys are in an advanced stage of preparation, pushing back the boundary of the mapped universe. Figure 1 gives an impression of the large-scale structure as we know it today, showing the galaxy positions in the CfA2 redshift survey and its southern counterpart, the SSRS2 survey (da Costa et al. 1994). Plotted are the right ascension versus the recession velocity of each of the 15,000 galaxies in the sample, having an outer boundary of the volume surveyed lying at a recession velocity of about 15000 km/sec. On the basis of these and other available surveys we can make a tour along the various structures to be found in our local niche of the universe.

### 3.2. INVENTORY OF NEARBY COSMIC STRUCTURES

Our immediate neighbourhood provides a reasonably representative sample of the universe, with a distribution that is evidently far from uniform. Our own Galaxy has two nearby companions – the Large and Small Magellanic Clouds – and together with the Andromeda galaxy M31 it dominates the so-called *Local Group* of galaxies, which consists of some 30 members and is a few Mpc in extent (Hodge, 1994). Further away, at a distance of approximately 17 Mpc, we encounter a prominent cluster of galaxies,



*Figure 1.* The galaxy redshift distribution (da Costa 1993), right ascension versus recession velocity, in opposite  $18^\circ$  thick slices (in the indicated declination range) in the northern CfA2 survey (see e.g. Geller & Huchra 1989) and the southern SSRS2 survey (da Costa et al. 1994). All galaxies with a magnitude brighter than  $m = 15.5$  are included. The northern slice includes the Coma cluster and the Great Wall. In the southern slice several wall-like structures can be seen forming a cellular-like structure (also see da Costa 1993). Figure kindly provided by L. da Costa.

the *Virgo cluster*. Its gravitational pull induces an infall velocity of a few hundred km/s of the Local Group towards the Virgo cluster. Although its proximity facilitates a detailed study of its structure, it is rather difficult to get an overall view of the cluster because of its large angular extent on the sky. Some 1300 members, and 500 additional possible members have been identified. A detailed analysis (Binggeli, Tammann & Sandage 1987) shows it to be a rather irregular structure with two pronounced concentrations, of which the major one contains the giant ellipticals M87 and M49. Several spatial and kinematic characteristics of the cluster suggest it to be a rather young structure, among others the fact that there is quite some substructure in the vicinity of the cluster core, which therefore must still be in the process of formation. A property that the Virgo cluster has in common with many other clusters is the strong spatial segregation, with the early type galaxies being considerably more concentrated to the cluster centres than the spirals and irregulars. Within the scheme of galaxy clustering and structure formation the dense and rich clusters of galaxies stand out as the structures that represent the fringe of objects that can still be considered individually distinguishable entities, in the sense of being the most massive fully collapsed objects in the universe. Wandering further out from the Virgo cluster, we find several substantially more impressive clusters within  $100h^{-1}$  Mpc of the Local Group. One of the most massive clusters we know of, the *Coma cluster* at a distance of around  $68.5h^{-1}$  Mpc, is located within this region (see fig. 2). It is one of the most regular, richest and best-observed clusters, containing many thousands of galaxies. Its regular spherically symmetric appearance and relative proximity made the Coma cluster into the canonical example of a rich and relaxed cluster, against which the characteristics of all other clusters were compared. However, recent work, especially concerning the X-ray emission associated with the hot intracluster gas that has been trapped in its potential well, has indicated that the cluster displays significant substructure (White, Briel & Henry 1993). The presence of such substructure is a strong indication that clusters are relatively young and dynamically active objects. A recent close examination of the galaxy distribution and kinematics of the Coma cluster confirmed this, showing that the cluster consists of a massive central cluster and a clearly distinguishable smaller subcluster that is just falling onto Coma, while the core itself consists of at least two subclumps that are already substantially disrupted and in the process of merging (Colless & Dunn 1995).

Also, the Coma cluster offers a detailed view of the relation of clusters to their spatial surroundings. Coma is not an isolated object. On the contrary, going outward from the cluster core we see a gradual transition of an increasingly flattened galaxy distribution into a strongly elongated fila-

*Figure 2.* Optical image of the central region of the Coma cluster. The cluster contains thousands of galaxies. The picture is obtained from the Digitized Sky Survey, based on photographic data of the National Geographic Society – Palomar Observatory Sky Survey (NGS-POSS), and was kindly provided by H. Böhringer.

ment, in particular when looking at the distribution of early-type galaxies (Doi et al. 1995). This filament extends from the NE to the WSW and embeds the Coma cluster into the surrounding large-scale structure. Since the publication of the results of the CfA survey (e.g. Geller & Huchra 1989) we know that the Coma cluster is in fact the strongest density concentration in the “Great Wall”, a gigantic and coherent plane-shaped assembly of galaxies, whose dimensions are estimated to be of the order of  $60h^{-1} \times 170h^{-1} \times 5h^{-1}$  Mpc. This structure can be clearly discerned in figure 1 as the dark and thick band of galaxies in the northern hemisphere, running all the way from the left to the right of the survey slice at a recession velocity of around 7,000 km/s. The Coma cluster itself is visible as the “Finger of God” in the centre, the elongated stripe pointing towards us

in the redshift map induced by the dispersion of peculiar velocities of its galaxies. More detailed studies reveal that besides the Coma cluster itself several other density enhancements can be recognized within this wall of galaxies. An example is the A1367 cluster, lying along the direction of the the filament emanating from the Coma cluster.

In fact, the relation and proximity to other clusters contains a lot of information on the presence and extent of even larger structures in which the clusters themselves are embedded. Numerous studies have shown that the distribution of rich clusters forms a useful and to some extent complementary tracer of the large scale matter distribution in the universe, certainly for scales exceeding twenty or more megaparsec (see Bahcall 1988). In particular the well-known catalogue of the most prominent clusters compiled by Abell (1958), though probably beset by some not completely understood artefacts, has been of great use in obtaining an idea of clustering on such scales. From Abell's catalogue as well as from some more recent compilations of clusters of galaxies we have learned that they have the tendency to group even closer together, relatively speaking, than galaxies themselves. Together with groups of clusters and a host of more loosely bound galaxies they appear to congregate in superclusters. These *superclusters* are huge, loosely-bound, non-virialized structures containing several to many rich clusters grouped in a usually highly flattened or elongated configuration. In comparison to galaxy clusters they represent far smaller overdensities, usually in the order of only a few times the average density of the universe, so that they did not yet have the time to collapse. Their structure and dynamics are therefore still closely related to the initial density fluctuations that gave rise to them, making them interesting probes, 'fossils', of the structure formation process. It actually took some time before the reality of these structures got accepted, and it was not until the review by Oort (1983) that they indeed got generally recognized as such.

Both our Local Group and the Virgo cluster are members of such a structure, the Local Supercluster, a huge flattened concentration of about fifty groups of galaxies in which the Virgo cluster is the dominant and central agglomeration. The Local Supercluster is in fact a modest specimen of its class, dominated by only one rich cluster. A far more prominent example of a supercluster, and in some sense more characteristic if one wishes to obtain an idea of their morphological variety, is the Perseus-Pisces supercluster (see fig. 3). Due to its relatively nearby distance of around  $55h^{-1}$  Mpc, its characteristic and salient filamentary geometry, and the main ridge's favorable orientation perpendicular to the line of sight, it has become one of the best mapped and studied superclusters. It is a huge conglomeration of galaxies that clearly stands out on the sky, just south of the plane of our Galaxy on the northern hemisphere. The boundary of

*Figure 3.* The Perseus-Pisces supercluster chain of galaxies. Separate two-dimensional views of the galaxy distribution in the northern region of the Pisces-Perseus region. The upper panel shows the sky distribution of all galaxies in the overall northern survey sample of Wegner, Haynes & Giovanelli (1993). The region believed to contain the Pisces-Perseus main ridge is outlined. The lower panel shows the two dimensional redshift distribution (right ascension-recession velocity  $V_0$ ) for galaxies in the ridge region highlighted in the upper panel. From Giovanelli & Haynes 1996, kindly provided by M. Haynes.

the supercluster on the northern side is formed by the filament running southwestward from the Perseus cluster, a majestic chain of galaxies of enormous proportions. It has a length of at least  $50h^{-1}$  Mpc and a width of about  $5h^{-1}$  Mpc. It even might be that the ridge extends out much further and has a total length of up to  $140h^{-1}$  Mpc, but obscuration by the Galactic Disk prevents firm conclusions on this point. Along the ridge we see a continuous arrangement of high density clusters and groups, of which the most notable ones are the Perseus cluster itself (Abell 462), Abell 347 and Abell 262. It happens to be one of the few large scale objects whose structure has been studied meticulously with the help of radiotelescopes,

mainly the Arecibo telescope. Giovanelli, Haynes & collaborators (see e.g. Wegner, Haynes & Giovanelli, 1993, and fig. 3) carried out an extensive and detailed analysis of the structure of the whole supercluster by mapping the positions of approximately 5000 galaxies in the region of the Pisces-Perseus supercluster. The determination of 21-cm redshifts for all late-type galaxies in the region is an important and typical aspect of their survey. Figure 2 gives an impression of the structure of the main Perseus-Pisces chain as obtained from this redshift survey (Giovanelli & Haynes, 1996). The upper panel shows the sky distribution of all the survey galaxies in the Perseus-Pisces region. The region believed to contain the PP main ridge is outlined. By plotting the sky position of the galaxies against their recession velocity  $V_0$ , the lower panel shows that it is indeed a real ridge in three-dimensional space. Notice that the Perseus cluster is recognizable in the lower panel as the “Finger of God” towards the left. From figure 3 we can in fact see that it is quite feasible to map large scale structures with the help of for example the SKA. Moreover, no longer would we be restricted to a relatively narrow redshift range and small area on the sky, so that mapping of structures like the Perseus-Pisces chain could be carried out to much higher redshifts and at virtually every location on a large fraction of the sky.

By now of the order 20 or so superclusters have been identified. Claims that there might be identifiable objects of an even larger size, in the order of several hundred Mpc, have not been substantiated. It is quite likely that in the local universe the Shapley concentration, the most impressive, and monstrous, supercluster complex that we know, represents the strongest density fluctuation on scales of  $25h^{-1}$ Mpc and larger. Covering an area of at least 15 degrees in radius on the sky and located at a distance of around  $150h^{-1}$ Mpc, some  $100h^{-1}$ Mpc behind the Hydra-Centaurus supercluster, it is the most massive concentration of clusters of galaxies in our local corner of the universe. The complex is dominated by a central region in which more than 20 rich clusters are crammed within a significantly flattened region of radius  $12.5h^{-1}$ Mpc around the extremely rich cluster Shapley 8. The total mass of this crowd of clusters has been estimated to be in the order of  $10^{17} M_\odot$  (see e.g. Quintana et al. 1995). Confined within such a volume this mass corresponds to an overdensity in excess of 2.4 times the average density of the universe, truly exceptional for that scale. Probably not surprising for a region so crowded and compact, it appears to be dynamically very active, with clusters merging at a high rate and with large deviations from the Hubble flow (Raychaudhury et al. 1991). When it turned out that the motion of our Local Group with respect to the universe is more or less directed towards the Shapley concentration, this obviously triggered speculations that its gravitational pull might be responsible for this peculiar motion, and that it therefore might be the ‘Great Attractor’

*Figure 4.* The Boötes void as revealed by the galaxy number space density in a sequence of five different recession velocity intervals in the direction of the Boötes constellation on the sky. The lowest contour represents a density equal to 0.7 of the cosmin mean, each higher contour represents a factor of 2 increase in density. Velocity ranges (km/s): (a) 7,000-12,000; (b) 12,000-17,000; (c) 17,000-23,000; (d) 23,000-29,000; (e) 29,000-39,000. Frame (b) clearly reveals a large void in the galaxy distribution, which turns out to be roughly spherical in outline. Figure from Kirshner et al. (1987).

whose existence was inferred from the peculiar velocity field in our local vicinity. Although this is certainly a possibility, given the fact that such a massive complex is located at such a nearby distance, present estimates are that it would only account for at best 25% of the total motion of the Local Group.

### 3.3. COSMIC VOIDS

Perhaps one of the most intriguing discoveries emanating from extensive redshift surveys has been the existence of large *voids* in the galaxy distribution, enormous regions, sometimes up to tens of megaparsec in extent, wherein few or no galaxies are found. The Boötes void in the KOSS redshift survey (Kirshner et al. 1981, 1987, see fig. 4) was the first void to attract the attention. It is an almost completely empty spherical region with a diameter

of around  $60h^{-1}$  Mpc and is considered to be the most typical example (fig. 4). Various redshift surveys covering large parts of the local universe have revealed that voids with sizes typically in the range of  $\sim 20 - 50h^{-1}$  Mpc (see e.g. Vogeley, Geller & Huchra 1991) are a common feature in the galaxy distribution. This leads to the important conclusion that voids must be an essential structural element in the universe, and recent evidence on the basis of a deep redshift survey (Bellanger & de Lapparent 1995) showed that this is the case out to redshifts of at least 0.5. Also, redshift surveys seem to suggest that the galaxy voids are generally associated with surrounding enhancements in the galaxy density, and probably these associated density excesses are stronger when the void is bigger. For example, the boundary on the near side of the Boötes void is formed by the Hercules supercluster, and on the far side by the Corona Borealis supercluster. A more detailed look at for example the Boötes void has made clear that voids are not necessarily completely empty. They should rather be considered as under-populated regions of space that often do contain some galaxies. The most systematic effort for finding galaxies within the realm of the Boötes void region itself is the HI survey by Szomoru et al. (1996). In total 34 galaxies were discovered in HI, bringing the total to 58 galaxies inside the void. Szomoru (1995) even speculated that some of these galaxies delineate tenuous filaments inside the void.

That voids in the galaxy distribution also correspond to voids in the matter distribution is an issue that has not yet been completely settled. In the general picture of structure formation structure forms by the growth of initially very small density fluctuations under the influence of gravity (see discussion below). Voids in the matter distribution will have formed out of the underdense regions in the initial matter distribution of the universe (see e.g. Van de Weygaert & Van Kampen 1993). Because there is less matter to slow down their cosmic expansion velocity, these regions correspond to a lower than average gravitational attraction. The matter in these protovoids therefore keeps on expanding with a higher velocity and consequently, with respect to the expanding background of the universe, starts to flow out of the underdense regions. Consequently, these protovoids will contain even less matter and induce an even lower gravitational attraction. This results obviously in an underdense region that becomes more and more empty with time. If one were to measure the peculiar motion of matter inside and near the edge of these voids, one would be bound to see the relative expansion of the void. In other words, effectively one would observe a ‘pushing’ influence of the void. Indeed, studies of peculiar motions of galaxies in the neighbourhood of voids produced evidence for this gravitational influence of voids (see e.g. Bothun et al. 1992; Dekel & Rees 1994). If voids indeed correspond to considerable perturbations in the matter distribution, they



will represent significant disturbances in the gravitational potential. Using this fact, a statistical study of their sizes could yield important constraints on the spectrum of initial perturbations (Blumenthal et al. 1992). Moreover, in that case the issue comes up whether their low density environment would in any way influence the process of galaxy formation. It might be that the formation of galaxies would be effectively suppressed so that they would even be more conspicuously empty in the galaxy distribution. Alternatively, the physical properties of the void galaxies may be different from the galaxies in less extreme and more average environments. According to some suggestions void galaxies are more likely to be dwarf galaxies or low surface brightness galaxies, whose gas supply did not get efficiently processed into stars. The implication would be that they have a relatively high HI content, implying that further insight may come from 21-cm line studies. The first indications from such a survey of the Boötes void by Szomoru et al. (1996) are that void galaxies are ‘normal’. More comprehensive and systematic studies will need the capacity to measure HI redshifts over an extended redshift range. A telescope like the SKA will therefore be almost essential in shedding more light on this issue.

### 3.4. THE COSMIC FOAM

Having made an inventory of the structures that we can find in the distribution of galaxies, we now turn to the issue of how it all fits together in a coherent picture. For this we return to the discussion of redshift surveys and to the redshift map in figure 1. A two-dimensional “slice”, a narrow  $6^\circ$  band of nearly  $120^\circ$  wide and to a depth of approximately  $15h^{-1}\text{Mpc}$ , was taken by the CfA group as an optimal survey geometry for the aim of studying the morphology of large scale structure. The CfA2 redshift survey consists of a set of such slices in which the redshifts of all galaxies with an apparent magnitude  $m < 15.5$  were determined. It was with the publication of the results of the first CfA slice, by de Lapparent, Geller & Huchra (1986), that we got to recognize that the galaxies are distributed in an intriguing *foamlike* or *bubbly* pattern, with under-populated regions (the *voids*) surrounded by *walls* and *filaments* (the superclusters), at whose intersections we find strong density enhancements in the form of *clusters* of galaxies (fig. 1). When further slices were added to the first slice, the basic picture of a ‘cosmic foam’ got confirmed, while the existence of the even larger structure got revealed, the *Great Wall*. So far the results for 4 slices, in total containing slightly more than 12,000 galaxies, have been published. An extension of this optical redshift survey on the southern sky by da Costa and collaborators (da Costa et al. 1994), the Southern Sky Redshift Survey extension (SSRS2) which consists of about 3600 galaxies

brighter than  $m = 15.5$ , displays a similar pattern of a void-filled universe with wall-like structures surrounding empty voids and even of a great wall (fig. 1, da Costa 1993).

### 3.5. THEORIES OF STRUCTURE FORMATION

Explaining how the described intricate structural patterns in the galaxy distribution have originated from the almost perfectly smooth early universe is evidently a challenging task for theories of structure formation. The issue of the origin of the galaxy distribution involves two closely related problems. The first one is to explain how structure has built up in the overall matter distribution, and the second one is when, how and where the galaxies have formed in the underlying ocean of matter. An overall theory of structure formation can only then be called complete and successful if it succeeds in explaining both issues from first principles and withstand comparison with, among others, the observed galaxy distribution. Although as yet we are still far away from achieving this goal, a general framework for the formation of structure is gradually emerging.

Firstly, we have to go back to the time at which we can see the earliest traces of structure in our universe. From the high degree of isotropy of the microwave background radiation we can infer that the universe was highly homogeneous at the epoch of recombination, when electrons and protons combined into hydrogen atoms. This occurred when the universe was a mere million years old and had cooled down to a temperature of  $\approx 3,500\text{K}$ , at a redshift of  $z \approx 1,300$ . Since they got last scattered by free electrons at that epoch, the MWB photons have been travelling uninterruptedly until they hit our telescopes. The slight variations in angular distribution of the measured temperature of the microwave photon bath must therefore contain direct information on the local circumstances at the surface of last scattering. It was only in 1992 that such tiny anisotropies in the microwave background temperature were actually found, when the COBE satellite detected small ripples in temperature with an amplitude in the order of only  $\Delta T/T \approx 10^{-5}$  (Smoot et al. 1992). While these anisotropies proved the existence of structures at the recombination epoch on very large scales in the order of a gigaparsec, it also showed how very low their amplitude is.

The finding of COBE is a remarkable confirmation of the general theoretical framework of ‘gravitational instability’ for cosmic structure formation. According to this theory the early universe was almost perfectly smooth except for tiny density variations with respect to the general background density of the universe and related tiny velocity perturbations with respect to the general Hubble expansion. Because slight density enhance-

ments exert a slightly stronger gravitational attraction on the surrounding matter, they start to accrete material from its surroundings as long as pressure forces are not sufficient to counteract this infall. In this way the overdensity becomes even more overdense, and their gravitational influence even stronger. The denser it becomes the more it will accrete, resulting in an instability which can ultimately cause the collapse of a density fluctuation to a gravitationally bound object. The size and mass of the object is of course dependent on the scale of the fluctuation. For example, galaxies are thought to have formed out of fluctuations on a scale of  $\approx 0.5h^{-1}\text{Mpc}$ , while clusters of galaxies have emerged out of fluctuations on a larger scale of  $\approx 4h^{-1}\text{Mpc}$ . The formation of voids fits in the same general scheme, having grown out of primordial underdensities in the matter distribution.

Providing the general framework, the gravitational instability theory needs lots of details to be filled in before it can be considered a complete theory. There is of course the issue of the amount of matter represented by a density fluctuation, as more massive fluctuations will collapse sooner. Given the amplitude of the fluctuations, their total mass is determined by the average cosmological density, parameterized by  $\Omega$ . The very low value of the amplitude of the primordial density fluctuations inferred from the COBE MWB measurements is a strong argument in favour of a high overall density of the universe. Otherwise, density fluctuations would simply not have had sufficient time to collapse on all the scales that nowadays are observed to exhibit so much structure. Also some other observational indications support a high value of  $\Omega$ , which has the important implication that most likely the major share of matter in the universe does not consist of familiar baryons and leptons but of one or more as yet unidentified species of ‘dark matter’.

The nature and amount of dark matter is also of substantial influence in determining the character of the initial density and fluctuation field, probably the most crucial issue in the structure formation saga. Rather than consisting of some isolated, well-defined and smooth density peaks and dips, each of its own particular scale, the density field can be thought of as a random superposition of fluctuations of various scales. It will therefore bear the character of a noise field, ‘a random field’, a random superposition of waves much like the surface of the sea at rough weather. Evidently, the waves with the largest amplitude will collapse first. The character of the density field evolution will then depend on the relative amplitudes of the different waves. One extreme case is that of small scale waves having by far the highest amplitude. They will collapse into virialized objects well before a larger scale perturbation, in which they are possibly embedded, starts to collapse. Consequently, we will see a hierarchical or ‘bottom-up’ build-up of structure, where small objects that formed first merge into larger

structures, which themselves merge to form galaxies, cluster of galaxies, and so on. The other extreme is that of the case in which there are only perturbations on large scales, with no contributions from smaller scales. In such a ‘top-down’ scenario the first emerging structures form through the collapse of those large scale perturbations. In the most popular versions of ‘top-down’ theories these objects would correspond to superclusters. Subsequently, smaller objects like galaxies have to form through the fragmentation of these collapsed large objects into smaller pieces, an as yet mostly ununderstood process in which non-gravitational gas processes play a key role.

The formation of anisotropic structural patterns in these random density fields is the consequence of an additional characteristic property of gravitational collapse. Overdensities, on any scale and in any scenario, always collapse such that they become increasingly anisotropic. At first they turn into a flattened ‘pancake’, later possibly followed by contraction into an elongated filament or by full collapse into a virialized clump like a galaxy or a cluster. This tendency to collapse anisotropically is caused by the intrinsic primordial flattening of the overdensity as well as by the anisotropy of the gravitational force field induced by the external matter distribution, i.e. by tidal forces. In the case of a pure hierarchical scenario the amplitude of large scale overdensities will be so low that they will not really have started their anisotropic collapse before the small scale overdensities have turned into high-density virialized clumps. Instead of appearing like a large coherent anisotropic structure the resulting large scale matter distribution will therefore more resemble a mere incoherent and shapeless density enhancement in the number of small clumps. On the other hand, in less extreme hierarchical scenarios large scale density fluctuations will have an amplitude high enough such that by the time small scale clumps have completely collapsed the large scale structure in which they are embedded will already have contracted substantially. In those cases we expect to see more or less coherent walls and filaments in which the small scale clumps stand out like beads on a string. Finally, in the most extreme ‘top-down’ case we will only see the anisotropic contraction of a large scale object like a supercluster. The resulting pattern will be one of a network of filaments and walls without any internal structure.

Evidently then, the decisive factor in determining the outcome of the evolution of a density field, and the structure of the universe, is the function that specifies the relative amplitude of the various ‘density’ waves. This function is called the *spectrum* of density fluctuations. The present standard view is that the very early universe produced a hierarchical spectrum of density fluctuations, the so-called Harrison-Zel’dovich spectrum. This primordial spectrum gets modified dependent on the nature of the matter

*Figure 5.* The evolution of the dark matter distribution in a computer simulation of the gravitational growth of structure in the standard Cold Dark Matter scenario ( $\Omega = 1$  and  $H_0 = 50 \text{ km/s/Mpc}$ ). At six different cosmic epochs, corresponding to redshifts  $z = 2.3, 1.5, 1.0, 0.7, 0.25$  and  $0.0$  (from left to right, top to bottom), the figure shows the particle distribution in the  $5h^{-1}\text{Mpc}$  thick central slice through the simulation box of size  $100h^{-1}\text{Mpc}$ . The simulation follows the evolution of a system of 262,144 particles in this box under the influence of their mutual gravity, assuming periodic boundary conditions. The N-body simulation was carried out with the P<sup>3</sup>M gravitational N-body code of Bertschinger (see Bertschinger & Gelb 1991)

content of the density field on which these fluctuations are imprinted. For example, if most matter in the universe would be dark matter in the form of massive neutrinos, the most typical species of hot dark matter, all fluctuations below the free-streaming scale of the almost relativistically moving neutrinos would be damped. This implies that the first perturbations that would collapse are on this scale, corresponding to that of present-day superclusters. In other words, the hot dark matter model is a top-down scenario. However, still the most popular scenario is the Cold Dark Matter scenario. In the CDM scenario the dark matter consists of some hitherto unknown species of collisionless particles having a negligible velocity dispersion. Consequently, these particles will not damp any primordial fluctuation. The only noticeable modification of the primordial spectrum is a turnover at a scale of around  $13/\Omega h^2$  Mpc caused by a suppressed growth of fluctuations in the epoch before matter takes over from radiation as the dominant component of the universe. Structure formation in the Cold Dark Matter scenario will therefore be the result of hierarchical clustering.

Figure 5 contains an illustration of structure formation in the Cold Dark Matter scenario. In a box with a size of  $100h^{-1}$  Mpc we set up a realization of a density and velocity fluctuation field specified by the CDM spectrum (for  $\Omega = 1.0$  and  $H_0 = 50$  km/s/Mpc). This initial density field is then sampled by a large number of discrete particles. Subsequently, we let the computer calculate how this particle distribution would evolve due to their mutual gravitational interaction. In six frames (from left to right, top to bottom) we show the particle distribution in a central slice through the simulation box at successive redshifts of 2.3, 1.5, 1.0, 0.7, 0.25 and 0.0, the present epoch. The frames evidently show a strong growth of structure on all scales. On small scales we see that earlier collapsed objects later merge into large clumps. Also noticeable is the emergence of a filamentary and wall-like pattern, which becomes particularly pronounced at later epochs due to the collapse of the corresponding density fluctuations and the constant flux of matter from lower density areas.

To give an idea of the dynamical background of the formation of these features in the large scale matter distribution we display in figure 6 the gravitational force, wrt. the mean expanding background of the universe, in combination with the corresponding particle distribution. In four panels, decreasing in size from  $100h^{-1}$  Mpc to  $15h^{-1}$  Mpc as we proceed from the top-left to the bottom-right frame, the gravitational force at regularly placed locations is represented by a vector whose size is proportional to the strength of the gravitational acceleration and which is directed along the force direction. In addition to the evident presence of strongly attracting massive clumps of matter, the two top panels present clear evidence for the ‘pushing’ influence of large empty voids. Focussing in on the condensed

*Figure 6.* The gravitational field in a N-body simulation of clustering in a standard Cold Dark Matter universe ( $\Omega = 1.0$ ,  $H_0 = \text{km/s/Mpc}$ ). The simulation is a 262,144 particle simulation in a  $100h^{-1}\text{Mpc}$  box with periodic boundary conditions. The 4 frames show, at different resolution, the particle distribution at redshift  $z = 0.0$  in 4 different  $5h^{-1}\text{Mpc}$  slices. The top-left box has a size of  $100h^{-1}\text{Mpc}$ , the top-right one of  $50h^{-1}\text{Mpc}$ , the bottom-left one of  $25h^{-1}\text{Mpc}$  and the bottom-right one of  $15h^{-1}\text{Mpc}$ . In addition to the particle distribution, each frame also displays the component of the gravity field in the box plane, representing it by a vector whose size and direction are proportional to the strength and direction of the gravity field at that position. The N-body simulation was carried out with the P<sup>3</sup>M gravitational N-body code of Bertschinger (see Bertschinger & Gelb 1991)

matter concentrations, we see that while at large scales the force field looks quite regular and displays less features than the matter distribution itself, at small scales it may display an interesting anisotropic infall pattern. Such an anisotropic force field is for example found near the agglomeration of clumps grouped along a filament in the bottom righthand frame. Within the theory of gravitational instability the induced velocities trace very well the gravitational field up to quite a late stage in the development of the corresponding density fluctuation field. This fact has spawn a large effort towards mapping the velocity field in the local universe, in the hope of being able to measure the gravity field, and thus directly the matter density fluctuations (see e.g. Dekel 1994 and Strauss & Willick 1995 for up-to-date reviews of this active and important cosmological field). As mentioned in section 2, the SKA could also yield significant advances in this scientific field through its ability to accurately determine peculiar velocities, through the Tully-Fisher relation, out to much larger distances than hitherto possible.

The structural pattern that is forming in the particle distribution in figure 5 bears a reasonably good resemblance to the one we observe in the galaxy distribution. On the other hand, careful analysis of available observational data suggest that available theories fail in explaining many aspects of galaxy clustering, in particular at scales in the order of  $100h^{-1}\text{Mpc}$ . There are for example substantial observational indications for the existence of more power at those scales than predicted by any of the presently popular structure formation scenarios. Notice though that the simulation in figure 5 indicates that also the CDM scenario yields structures on a scale of  $100h^{-1}\text{Mpc}$ , the size of the box, in an advanced state of development. However, comparison of theoretical models with the galaxy distribution on those scales is quite hampered by it being near the limit of the present-day galaxy surveys. More definitive conclusions can therefore not be expected without exploring the galaxy clustering out to greater depths in the universe. In fact, studies of the galaxy distribution at much higher redshifts have the additional virtue that they would possibly also yield information on the evolution of galaxy clustering from earlier epochs onward. As yet the information on clustering at earlier epochs can at best be called scarce.

### 3.6. EXPLORING STRUCTURES AT GREATER COSMIC DEPTHS

While these theoretical reasons provide an important motivation for trying to push the galaxy redshift survey limits further out, the need to do so is also suggested by the present galaxy surveys themselves. One aspect that is quite clear from relatively shallow surveys like the CfA2 survey is that the largest structures that are seen are comparable in size to the survey volume. In other words, we do not yet have a representative sample of



*Figure 7.* An example of a deep pencil beam redshift survey, showing the redshift distribution of galaxies out to a distance of  $1200h^{-1}\text{Mpc}$  towards the south Galactic pole (negative velocities) and the north Galactic pole (positive velocities). Plotted is the number of galaxies in  $10h^{-1}\text{Mpc}$  bins. To be precise, the figure is a combination of several very narrow pencil beam redshift surveys, comprising fields of 5 to 20 arcminutes. The black bars represent the number of galaxies in the original survey of Broadhurst, Ellis, Koo & Szalay (1990). The superposed dotted bars represent more recent extensions of and additions to the original (1990) survey. The continuous curve at the background is the survey selection function, which combines the effects of the different geometries and apparent magnitude limits of composite survey beams. Figure was kindly provided by Alex Szalay.

structures in the universe and we have not yet reached the scale at which the universe can be considered truly homogeneous. A practical problem is that it becomes more and more difficult to probe the galaxy distribution deeper in the universe as the determination of galaxy redshifts demands increasing amounts of telescope time because the galaxies become rapidly fainter with redshift. In order to overcome the requirements of depth, completeness, and limited telescope time several different survey strategies have been defined. One of the first attempts to go to very high redshifts restricted the survey to a very narrow angle on the sky. These surveys acquired the descriptive name of ‘pencil-beam’ survey. A very striking redshift distribution was found in the pencil-beam redshift survey of Broadhurst et al. (1990). Going out as far as  $2,000h^{-1}\text{Mpc}$  they found an apparent regularity in the galaxy distribution, huge spikes separated by gaps with a size of  $\sim 128h^{-1}\text{Mpc}$  (compare fig. 7, an extension of the original Broadhurst et al. 1990 pencil beam redshift survey). It is rather unlikely that such large gaps correspond

to equally large completely empty voids. The presence of empty voids of that size would put tight constraints on theoretical models of structure formation, and indeed would probably be in conflict with the smoothness of the microwave background radiation as only one such structure would be expected within the whole visible universe (Blumenthal et al. 1992). It is therefore more likely that the survey only picked up the most conspicuous features along its line of sight while its very small effective beam width made it miss the smaller voids and structures in front and in between the big structures. Such a view is certainly supported by a comparison with the CfA and SSRS2 survey. It shows that the first spikes in the pencil beam survey correspond to the ‘Great Walls’ on both sides. Moreover, recent results of the equally deep but more completely sampled redshift survey by Bellanger & de Lapparent (1995) shows that the characteristic void-filled morphology of the CfA2/SSRS2 survey persists out to high redshifts, with voids having similar diameters. Maybe the Broadhurst et al. (1990) observation is suggestive of the existence of ‘supervoids’, large underdense regions with a size of  $\approx 130h^{-1}\text{Mpc}$  that are bounded by great walls and that do contain smaller voids, with sizes of  $20 - 50h^{-1}\text{Mpc}$  in their interior. If the local universe indeed testifies of such a ‘void hierarchy’ it may well mean that we are living in an underdense quarter of the universe.

Another attempt to probe very far out is that of the sparse surveys. Instead of measuring all the galaxy redshifts in a specific volume of space, only a fraction of the galaxy redshifts is measured, ideally by choosing the survey galaxies completely at random. They do not show striking structural features in as much detail as the dense optical surveys of CfA2 and SSRS2. However, they are excellent for inventarisation of structures on large scales and for producing maps of the galaxy density over these scales. An interesting example of such a survey is the QDOT survey (Saunders et al. 1991, QDOT is an acronym listing of the involved institutes, Queen Mary-Durham-Oxford-Toronto). The goal of the QDOT survey was to probe all structures on scales larger than  $\sim 10h^{-1}\text{Mpc}$  within a certain distance of our Local Group. For this purpose the survey galaxies were selected on the basis of the IRAS catalogue, because it provides such a well-defined and uniform sample of galaxies covering as much of the sky as possible. Important in this sense is the fact that the IRAS fluxes are little affected by extinction by the disk of our Milky Way. By its nature the IRAS catalogue already defines a sparse sample, as only a fraction of galaxies are represented, mainly late-type galaxies. From this catalogue QDOT selected 1 in every 6 galaxies that have a  $60\mu\text{m}$  flux brighter than 0.6 Jy, so that it probes out to an effective depth of  $200h^{-1}\text{Mpc}$  over the largest fraction of the sky. The QDOT survey has been very useful in identifying large density enhancements and depressions within this volume, revealing the existence

of several previously unidentified voids and galaxy concentrations besides previously known voids and superclusters such as the Persues-Pisces and Coma supercluster.

A third alternative to shortcut the effort to obtain information on clustering on very large scales is by concentrating on objects that are intrinsically more sparsely distributed than galaxies, that are related to certain aspects of the large scale matter distribution, and that can be detected out to much higher distances. For example, one could look at the clustering of quasars or other active galactic nuclei. However, their position in and relationship to the large scale matter distribution is still not very clear. Far more promising is to look at the high-density peaks of the galaxy distribution, i.e. at the rich clusters of galaxies. With their low space density and large mean separation one could compare their tracing of large scale structure with the way mountain peaks trace a mountain range (Bahcall 1988). The Abell catalogue of 1682 rich clusters, selected from the Palomar Sky Survey plates, is more or less complete to a redshift of  $z \sim 0.2$ . However, most results on the large scale distribution of clusters have been confined to a redshift  $z < 0.1$  ( $\approx 300h^{-1}\text{Mpc}$ ), based on a complete subsample of 104 nearby rich Abell clusters for which the redshifts have been determined. In the meantime deeper and more objectively defined cluster redshift samples are being compiled. Probably the catalogues selected on the basis of the cluster's X-ray luminosity are most promising for probing very deep into the universe (see Böhringer 1995). Clusters contain a large quantity of hot ( $10^7 - 10^8$  K) tenuous intracluster gas that strongly emits in X-rays. The total X-ray luminosity is a direct measure of the total mass of the cluster because of its direct relation to the depth and extent of the cluster potential well. Selected on the basis of the best available X-ray imaging survey, the ROSAT All Sky Survey, the ESO Redshift Survey of ROSAT clusters will yield upon completion the best defined and deepest cluster catalogue available, its  $\sim 700$  southern clusters probing out to an effective depth of  $\sim 600h^{-1}\text{Mpc}$  (Guzzo 1995). It might even be possible to explore scales up to  $1,000h^{-1}\text{Mpc}$  ( $z \sim 0.3$ ) on the basis of the in total 4000 to 5000 clusters expected in the ROSAT survey.

### 3.7. THE NEW AND DEEP GALAXY REDSHIFT SURVEYS

While pursuing alternatives and shortcuts to complete galaxy redshift surveys is an efficient way to obtain some specific information on super large scale clustering, their information content cannot compete against fully and uniformly sampled galaxy redshift surveys. Fully sampled galaxy surveys will trace much better anisotropic structures like filaments and walls. In addition, a fully sampled galaxy redshift survey will yield an optimal dy-

namic range of scales over which the density fluctuation spectrum can be determined.

A first effort towards an extension of galaxy redshift surveys like CfA2 and SSRS2 is the Las Campanas Redshift Survey. This survey includes redshifts of 26,000 galaxies with a mean redshift of  $z \sim 0.1$ , and consists of six slices covering an area of  $\approx 700$  square degrees. Although the Las Campanas redshift survey has some peculiar sampling characteristics, it is well suited for measuring the galaxy density fluctuations at scales in the order of  $100h^{-1}\text{Mpc}$ . Recently, Landy et al. (1996) determined the power spectrum on scales between 30 and  $200h^{-1}\text{Mpc}$  and found that there is a strong peak at a scale of  $\approx 100h^{-1}\text{Mpc}$ . Furthermore, they identified this peak to correspond to numerous walls and voids visible in the survey. This result indicates that structures similar to the ‘Great Wall’ in the CfA survey and the Boötes void are common features of the local universe, and it lends support to the claim of excess power on these scales inferred from the deep pencil beam survey of Broadhurst et al. (1990).

Two truly ambitious survey projects are at the moment in an advanced state of preparation, both meant to probe the galaxy distribution out to scales of  $\sim 1000h^{-1}\text{Mpc}$ . The largest and most comprehensive galaxy survey is undoubtedly the Sloan Digital Sky Survey, run by a consortium of U.S. institutions (see Gunn & Weinberg 1995 for an extensive discussion). For this redshift project a special, dedicated 2.5-meter telescope has been built with a corrected field of  $3^\circ$ . Its instrumentation will include a large multi-CCD camera and two double fibre spectrographs of each 320 fibres. The galaxy survey will cover a quarter of the sky in the northern Galactic cap, and consists of two major parts. The first is a large CDD photometric survey producing images in five colours to a limiting magnitude of  $\approx 23$ , yielding something like  $\sim 5 \times 10^7$  galaxies. From this photometric survey all galaxies brighter than magnitude 18 will be selected, and of these  $\sim 10^6$  galaxies the redshift will be determined, corresponding to a mean redshift of  $z \approx 0.1$ . In addition the survey will yield the spectroscopy of  $\sim 10^5$  quasars, while there will also be a repeated imaging of galaxies in a  $200\text{ deg}^2$  strip in the southern Galactic cap. It need no saying that the amount of information that the Sloan survey will produce is truly mind-boggling. Mapping the large scale galaxy distribution will only be one of the applications, be it one of the more important ones. To give an idea of how a  $6^\circ$  slice through the Sloan survey would look like, figure 8 shows a mock galaxy catalogue extracted from a large 54 million particle N-body simulation of a low-density ( $\Omega = 0.4$ ,  $\Lambda = 0.6$ ) CDM model (from Gott et al. 1996, also see Gunn & Weinberg 1995). The slice contains 66404 galaxies, 6.6% of the number expected over the full area of the northern Sloan survey.

A complementary survey is the Anglo-Australian 2 degree Field (2dF)

*Figure 8.* A simulated slice,  $6^\circ$  by  $130^\circ$ , through the SDSS redshift survey of the north Galactic cap. Galaxies are plotted at the distance that would be inferred from their redshift, so cluster velocity dispersions create “fingers of God” that point towards the observer. The slice contains 66404 galaxies, 6.6% of the number expected over the full area of the northern survey. This mock catalog is drawn from a large N-body simulation of a low-density ( $\Omega = 0.4$ ,  $\Lambda = 0.6$ ) CDM model. From Gott, Weinberg, Park & Gunn 1996, also see Gunn & Weinberg 1995.

redshift survey (see Lahav 1995), which will be carried out with the 4 metre Anglo-Australian Telescope and its 400-fibre spectroscopic facility covering a  $2^\circ$  field of view. It will produce the redshifts for around 250,000 galaxies brighter than magnitude 19.5, selected from the APM survey. In total it will cover  $\sim 1,700$  square degrees on the sky, and have a median redshift of  $z \sim 0.1$ .

It is clear that a succesfull operation of the Sloan survey and the 2dF survey will be a giant step towards understanding structure formation. Obviously it will yield a large amount of information on density fluctuations on scales between  $30h^{-1}\text{Mpc}$  and  $1000h^{-1}\text{Mpc}$ , allowing an accurate determination of the density fluctuation spectrum in between those scales as well as a better assessment of the structure of the foam-like patterns in the

galaxy distribution and the interrelationship between its constituents, the galaxy walls, filaments, clusters and voids. Moreover, hopefully this will also settle the issue at what scale the universe more or less reaches ‘homogeneity’. Besides these issues concerning the structure in the universe there is also a plethora of other, related, spin-offs. The peculiar motions of galaxies induced by the matter fluctuations leave their mark in the form of redshift distortions in the survey maps, and from these distortions it is possible to extract accurate estimates of  $\Omega$ , as well as to get a good idea of the biasing of the galaxy distribution with respect to the matter distribution. Moreover, just the five-colour photometric survey alone will already contain a wealth of information on the evolution of galaxies.

Although these ambitious projects will provide us with these overwhelming amounts of information on clustering, it still concerns the local universe out to redshifts of  $z \sim 0.1 - 0.2$ . Going out to truly gigantic scales corresponding to  $z \sim 1$  and larger would be a logical follow-up, even more so as that will allow for the first time exploration of the evolution of structure (compare e.g. fig. 5,  $z \sim 0.1 - 0.2$  is even more recent than the epoch of the last but not least frame). Following the specifications for the SKA and the discussion in section 2 it appears that to achieve that goal the Square Kilometer Array will be THE instrument of choice !

#### 4. Neutral hydrogen in the local universe

As discussed in section 2, in the local universe essentially all atomic hydrogen is associated with galaxies; there is no evidence for a major intergalactic component of HI. By looking at the HI content of galaxies around us one can therefore determine the ‘mass-spectrum’ of atomic hydrogen clouds, and this mass spectrum can serve as a basis for determining the number of ‘detectable’ galaxies. Table 1 gives the mass spectrum of HI clouds in the local Universe as determined by Briggs (1990). For a given  $S/N$  ratio, each successive line in this table requires an increase in observing time of a factor of 10. On the basis of table 1 one would conclude that it does not make much sense to try to detect galaxies with  $M_{HI} < 3 \times 10^9 M_\odot$ , because the gain in the number of galaxies is small with respect to the increase in observing time.

As an illustration of the enormous potential of a SKA for studies of the large-scale structure in the Universe, consider galaxies out to a redshift of 0.3 (*i.e.* six times farther than the limit in fig. 1). Using a Hubble constant  $H_0$  of 100 km/sec, the distance out to this redshift is about 750 Mpc. The number of galaxies inside this volume with a mass of atomic hydrogen larger than say  $3 \times 10^9 M_\odot$  is about  $1.3 \times 10^7$ , which corresponds to a surface

$\log M_{HI}/M_{\odot}$	Number per 1000 Mpc <sup>3</sup> with HI mass $> M_{HI}$
10.5	0.002
10.0	0.99
9.5	7.1
9.0	20.6
8.5	42.
8.0	73.
7.5	115.
7.0	174.

TABLE 1. Cumulative number density of galaxies as a function of HI mass (Briggs 1990).

density of 300 galaxies per degree<sup>2</sup>. A quick estimate of the observing time needed to measure line profiles for these galaxies can be made by comparing the SKA at  $z = 0.3$  with the VLA at  $z = 0.03$ . The performance of a SKA at  $z = 0.3$  should be better or similar to that of the VLA at  $z = 0.03$ , because the factor of about 80 in collecting area improvement makes up for the increase in distance by a factor of 9. Assuming a profile width of 120 km/sec we have  $5 \times 10^8 M_{\odot}$  of HI per channel of 20 km/sec. Combining this with a sensitivity of the VLA of 1.1 mJy per resolution element per hour of integration time (one  $\sigma$ ), a column density of HI of  $5 \times 10^8 M_{\odot}$  per resolution element results in a  $12 \sigma$  detection in one hour. Thus, in one hour one can obtain excellent lineprofiles with a SKA of galaxies with  $3 \times 10^9 M_{\odot}$  of HI at  $z = 0.3$ . This translates into 7000 line profiles per square degree per day. If one is not interested in the detailed shapes of the profiles of the spectral lines, but only in redshifts, the observing time may be reduced by an order of magnitude. In other words, of order 100000 redshifts could be obtained per day out to  $z = 0.3$  with a primary beam of one square degree. The efficiency of a SKA in measuring redshifts drops rapidly with increasing redshift, but as discussed in section 5, it will be possible to reach redshifts around 3 or 4.

## 5. Cosmological requirements for the SKA: its performance at high $z$

The issues related to the detection of neutral hydrogen at low and high redshifts differ fundamentally. At low  $z$  essentially all galaxies will be spatially resolved and the main questions are what column densities of HI can be reached in a given amount of observing time, say one day, and how far one can penetrate into the low end of the HI mass function given in table 1. At high  $z$  on the other hand the main question is whether galaxies at the upper end of the HI mass function ( $M_{HI} > 10^9$  to  $10^{10} M_\odot$ ) can still be detected at all, *i.e.* in say 100 days of observing time. This difference comes about because, depending on the adopted cosmological model, the observing time needed to detect a given HI mass increases at least with the fourth power of  $z$  (the total flux in an emission line decreases faster than with  $z^2$  and a decrease in the fluxdensity can only be beaten by increasing the observing time quadratically). This indicates that for a given HI mass there will exist a fairly sharp cut-off in  $z$  beyond which HI clouds can no longer be observed, simply because the observing time becomes excessive.

In section 4 we have argued that it may be possible to obtain redshifts at a rate of  $10^5$  per degree<sup>2</sup> per day out to  $z \simeq 0.3$ . But the real challenge of the SKA lies at redshifts of one and beyond. According to Braun (1995), the HI line sensitivity of a SKA at  $z = 1$  is close to  $1.0 \times 10^8 M_\odot$  ( $1\sigma$ ) per 24 hours observing time, per frequency channel of 100 kHz ( $\simeq 20 \text{ km/sec}$ ) for  $\Omega = 1$ . This number confirms our estimate in section 2, which translates into  $0.9 \times 10^8 M_\odot$  ( $1\sigma$ ). For a line profile one needs say  $3\sigma$  detections in 6 adjacent channels. The amount of hydrogen per galaxy must therefore be at least  $2 \times 10^9 M_\odot$ . Determining redshifts does not require an entire day however: a  $5\sigma$  detection, corresponding to 2 hours, would probably suffice. In table 2 we list the observing time needed for measuring redshifts as a function of  $z$  for a galaxy with  $2 \times 10^9 M_\odot$  of HI per resolution element for  $q_0 = 0.1$  and  $q_0 = 0.5$ .

Let us now try to estimate the number of galaxies inside a spherical shell at a given redshift that might be detected by a SKA. This estimate will be based on the mass function of HI in galaxies given in table 1. It is of course unlikely that this mass function will be applicable at high redshift. At least three processes affect this mass function. The conversion of gas into stars in the course of time will increase the average HI content of galaxies with increasing look-back time. At present the HI content of most of the brighter galaxies does not exceed 5 to 10 % of the stellar mass. Thus, at high  $z$ , depending on the star formation history, the typical HI mass per galaxies could be much higher than it is now. On the other hand, accretion



$z$	$q_0 = 0.1$	$q_0 = 0.5$
1.0	0.17	0.083
1.5	1.4	0.50
2.0	6.6	1.8
2.5	22.	5.0
3.0	62.	11.
3.5	148.	23.
4.0	316.	41.

TABLE 2. Observing time needed for redshift determinations for galaxies with  $2 \times 10^9 M_\odot$  of HI (in days).

of HI onto galaxies, the merging of small galaxies into bigger ones, and the higher level of ionising radiation at high  $z$ , act in the opposite sense. Little is known about these processes. Accretion and merging could be quite important because the faint end of the galaxy luminosity function is known to evolve strongly.

In table 3 we estimate the number of detectable galaxies per square degree. Following Braun (1995), this estimate is based on a receiver with 1280 channels. Using channels of 100 kHz ( $\simeq 20$  km/sec) at the emission frequency of 1421 MHz, results in the redshift interval given in the second column of table 3. In practice the situation is more favourable than our estimate indicates because the primary beam of the telescope will open up at larger wavelengths. For a primary beam of one degree<sup>2</sup> at 21 cm, the number of galaxies per beam is a factor of  $(1+z)^2$  larger than that given in the table.

At large redshifts it will in general not be possible to resolve the detected objects spatially. However, it is extremely important that the surroundings of galaxies can be mapped at least coarsely, with a linear resolution of say 5 kpc, in order to allow a study of the building up of galaxies from smaller units. For a large range in redshifts 5 kpc corresponds to an angular resolution of about one arcsec, and the resolution elements of a SKA should certainly be no smaller than this. The spectral resolution needed for cosmological studies is modest. With channels of about 20 km/sec wide one can obtain rough information on the shape of the line profiles and redshifts can be determined with an accuracy of a few km/sec.

$z$	$z$ interval	$N_{gal} \ q_0 = 0.1$ $\times 1000$	$N_{gal} \ q_0 = 0.5$ $\times 1000$
1.0	0.92 – 1.10	5.8	3.8
1.5	1.39 – 1.62	14.	7.8
2.0	1.87 – 2.14	27.	13.
2.5	2.35 – 2.67	46.	19.
3.0	2.83 – 3.19	69.	26.
3.5	3.31 – 3.72	99.	33.
4.0	3.79 – 4.24	134.	42.

TABLE 3. Estimated number of galaxies with  $M_{HI} > 3 \times 10^9 M_\odot$  per square degree.

In summary, studies of HI at high redshifts demand the following:

- Primary beam: as large as possible. Beams smaller than about one square degree will degrade the potential of the SKA for studies beyond  $z = 1$  substantially.
- Resolution elements: about one arcsec.
- Spectral resolution: channels of about 20 km/sec (at emission).

## Acknowledgements

We wish to thank Hans Böhringer, Frank Briggs and Luiz da Costa for encouraging and useful discussions. We are very grateful to Luiz da Costa, Hans Böhringer, Martha Haynes, Alex Szalay and David Weinberg for providing respectively figure 1, figure 2, figure 3, figure 7 and figure 8. In addition, we are grateful to J.R. Gott for the permission to use figure 8. Also we wish to acknowledge Doris Neumann for critically reading the text and for many helpful comments and suggestions, and Ed Bertschinger for providing the P<sup>3</sup>M N-body code. RvdW is supported by a fellowship of the Royal Netherlands Academy of Arts and Sciences. He also acknowledges the hospitality of the Max-Planck-Institut für Astrophysik in Garching, Germany, where part of this contribution was written.

The Digitized Sky Survey, used to obtain figure 2, was produced at the Space Telescope Science Institute under US Government grant NAG W-2166, and is based on photographic data of the National Geographic Society – Palomar Observatory Sky Survey, funded by a grant from the National Geographic Society to the California Institute of Technology.

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